

Power System State Estimation Distributed Computing Technique

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Abstract: — The growth of modern electric power system is phenomenal. In this era of automation, Electric companies depend more on SCADA based system. The main challenge is to solve large and complex non-linear equation with huge measured data in a very short time. The various techniques discussed in the literature are having their own limitations. A few such methods are Two level state estimation by M.Y.Patel, A.A.Girgis et al [2], Bahgat A. , Sakr [7], Fuzzy logic by Jeu-Min Lin, Shyh-Jier Huang et al., [3], Dynamic State Estimation by S. J. Huang and K. R. Shih [4], S. K. Sinha and J. K. Mandal, [5] and G. Durgaprasad, S. S. Thakur, [6], Multi-level state estimation by Mofreh, M. Salem, McColl et al...[9].

This paper presents an innovative state estimation technique without compromising the existing Newton-Raphson state estimation mathematical model. A node/bus along with its connected nodes/buses is called "Node Area". This node area can be treated as an independent entity provided sufficient measurements at each node area and adequate communication between the processors of each node-area are made available. The growing smart grid application to power system makes it suitable for this unique Node Area level of state estimation. This paper presents complete analysis of Node Area state estimation technique along with its computational time and comparison with the conventional Integrated State Estimation (ISE).

Index Terms—SE- State Estimation, WLS – Weight Least Square, NR – Newton-Raphson, ISE – Integrated State estimation, NASE – Node Area State estimation , NA – Node Area- A node along with its connected Node is referred as Node Area, H_1 to H_{12} are the sub set of Jacobian metrics 'J'.

1 INTRODUCTION

The Electrical power system is large and complex. Conventionally top down approach is adapted by the numerical solutions to solve the power system state estimation problem. In top down approach the whole power system network is considered as a single entity which involves huge measured data and large number of state variables resulting in increased complexity. Instead of top down approach the bottom-up approach for the same problem reduces the computational complexity. This is due to huge reduction in measurement vectors and state variables at Node Area level. The bottom-up approach can be applied by grouping of measurement vector at node area level. All the interconnected node area together forms whole network. Each node area can be treated as an independent entity by appropriately modifying the NR algorithm of electric power system state estimation. It is not necessary to divide the network in its physical plan in this modification. The conventional NR mathematical solution is procedure oriented iterative technique. Before understanding the modification of the existing Newton-Raphson state estimation it is necessary to review the procedural steps involved in the existing NR method. The brief insight of NR method is given below [1].

1.1 Newton-Raphson State Estimation –Review

The Newton-Raphson final equation is $A \Delta x = b$ by applying the tylor series to the nonlinear equations of power sys-

tem, following equations is derived. Here, the error vector includes nonlinear vector function $f(x)$. In order to estimate x , an initial value x_0 is assumed and x_0 is up-dated after iteration till convergence.

$$\begin{aligned} (J_0^T W J_0) \Delta x &= J_0^T W \Delta z \\ \text{Let } A &= (J_0^T W J_0) \quad \& \quad b = J_0^T W \Delta z \\ A \Delta x &= b \end{aligned} \quad (1)$$

$$\text{No of state variables} = (2 \cdot n - 1)$$

$$n = \text{number of network nodes, } 1, 2, \dots, n.$$

$$\text{Total no of measurements} = m$$

State variables V_i & δ_i = Voltage magnitude & phase angles
State variables

$$[x]^T = [\delta_1, \delta_2, \dots, \delta_{n-1}; v_1, v_2, \dots, v_n].$$

These measurements may include one or all quantities such as $[z(\text{measured})]^T = [P_i, Q_i, p_{ij}, q_{ij}, V_i, \delta_i]$

P_i, Q_i = Real & Imaginary part of injected power respectively

p_{ij}, q_{ij} = Real & imaginary part of line flows respectively

$\Delta z = z(\text{measured}) - z(\text{calculated})$

Dimension of Jacobian matrix = $m \cdot (2n - 1)$

'W' is the diagonal weigh matrix of the order of $(m \cdot m)$

This is a set of linear equations, if higher order terms of the Taylor expansion of $f(x)$ were really negligible; the solution yields the correct 'x'. The Jacobian J is itself a function of x. The state variable vector x can be obtained by solving the equation $[A \cdot \Delta x = b]$ iteratively.

The vector x should therefore be changed accordingly after every iteration till the convergence is obtained. $x^{c+1} = x^c + \Delta x^c$ ('c'-iteration count). Elements of Jacobian are derived from injected power and line flow equations.

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$$\begin{bmatrix} \Delta P_i \\ \Delta Q_i \\ \Delta p_{ij} \\ \Delta q_{ij} \\ \Delta v_i \\ \Delta \delta_i \end{bmatrix} = \begin{bmatrix} H_1 & H_2 \\ H_3 & H_4 \\ H_5 & H_6 \\ H_7 & H_8 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta v_i \end{bmatrix} \quad (2)$$

2 MODIFIED NR STATE ESTIMATION

As given in the multi-processing technique by H.N. Udupa, Dr. H.R.. Kamath [1] Jacobian Parallel State Estimation for node-wise grouping can be presented as shown below.

$$J_j^T W_{jj} J_j = A_j \text{ \& } J_j^T W_{jj} \Delta z_j = b_j \text{ for } j\text{th measurement} \quad (3)$$

Where $A = [A_1 + A_2 + A_3 + \dots + A_m] \dots$ (4) and

$$b = [b_1 + b_2 + b_3 + \dots + b_m] \dots (5)$$

For n^{th} Node Area measurements Jacobian relation is as follows

$$\begin{bmatrix} \Delta P_n \\ \Delta Q_n \\ \Delta p_{ij}^n \\ \Delta q_{ij}^n \\ \Delta v^n \\ \Delta \delta^n \end{bmatrix} = \begin{bmatrix} H_1^n & H_2^n \\ H_3^n & H_4^n \\ H_5^n & H_6^n \\ H_7^n & H_8^n \\ H_9^n & H_{10}^n \\ H_{11}^n & H_{12}^n \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta v_i \end{bmatrix} \quad (6)$$

(The subscript 'NAi' refers to 'ith' node area)

$\Delta p_{ij}^r, \Delta q_{ij}^r$ is the real and reactive line flow measurements between the connected nodes of r^{th} node area and $\Delta v^r, \Delta \delta^r$ is the voltage and angle measurements between the connected nodes of r^{th} node area.

()_{NAi} The subscript 'NAi' refers to 'ith' node area

A_r^P is the sub set of matrix 'A' for the injected Real power measurement taken at 'r'th node/bus. Similarly, A_r^Q is the sub set of matrix 'A' for the injected Reactive power measurement taken at 'r'th node/bus and likewise for other measurements.

$$\begin{aligned} NA_r &= \left\{ \left(A_r^P + A_r^Q + \sum A_r^{pij} + \sum A_r^{qij} + A_r^v + A_r^\delta \right) \right\} \\ &= J_{NAr}^T W_{NAr} J_{NAr} \quad (7) \\ b_{NAr} &= \left\{ \left(b_r^P + b_r^Q + \sum b_r^{pij} + \sum b_r^{qij} + b_r^v + b_r^\delta \right) \right\} \end{aligned}$$

$$= J_{NAr}^T W_{NAr} z_{NAr} \quad (8)$$

$$\sum_{j=1}^n (A_{NAj}) \Delta x = \sum_{j=1}^n (b_{NAj}) \quad (9)$$

$$\sum_{j=1}^m (A_j) = \sum_{j=1}^n (A_{NAj}) = A \quad (10)$$

$$\sum_{j=1}^m (b_j) = \sum_{j=1}^n (b_{NAj}) = b \quad (11)$$

3 NODE AREA STATE ESTIMATION (NASE)

A node/bus alone with its connected node/bus is called as Node Area. It is clear from equation (3), (4) & (5) that 'a' measurement can be computed at a time to form the corresponding sub-set of matrix A and b. By grouping all the measurements of a particular Node Area and computing as per equations (3) to (5) will yield 'A' and 'b' corresponds to that Node area. If the number of measurements available at the Node Area is greater than the number of state variables of that Node Area then 'A' and 'b' can be solved to find the Node Area state variables. The number of state variables at a Node Area depends on the total number of nodes of Node Area. This technique helps reducing the large single problem into small independent sub-task. The formulation and NASE algorithm are detailed out in the following section.

3.1 Formulation

The k^{th} Node area measurements many include $[P_k, Q_k, p_{ij}^k, q_{ij}^k, v^k, \delta^k]$. If sufficient measurements are made available at each node area, from equation (6) and (9) it can be written as

$$\begin{aligned} (J_{NAk}^T W_{NAk} J_{NAk}) (\Delta x_i)_{NAk} &= (J_{NAk}^T W_{NAk} z_{NAk}) \\ A_{NAk} (\Delta x_i)_{NAk} &= b_{NAk} \quad (12) \end{aligned}$$

where 'NAK' refers to k^{th} node area and $(\Delta x_i)_{NAk}$ is the state vector corresponds to k^{th} node area.

$(x_i^{c+1})_{NAk} = (x_i^c)_{NAk} + (\Delta x_i^c)_{NAk}$; where 'c' is the iteration count and 'k' = (1, 2, ..., n). The equation (1) is for whole network whereas the eq (12) is for k^{th} node area.

3.2 NASE Block diagram: -

The figure 3.1 shows the block diagram representation of data acquisition and communication system at a Node/Bus. Using this new technique state estimation can be carried out at each bus/node (NASE). The point to be noted here is that a node will be connected to another one or more nodes in the network. Hence, 'a' node will have more than one estimated values. After performing Node level estimation each node can communicate between the connected nodes and thereby compute the final estimation.

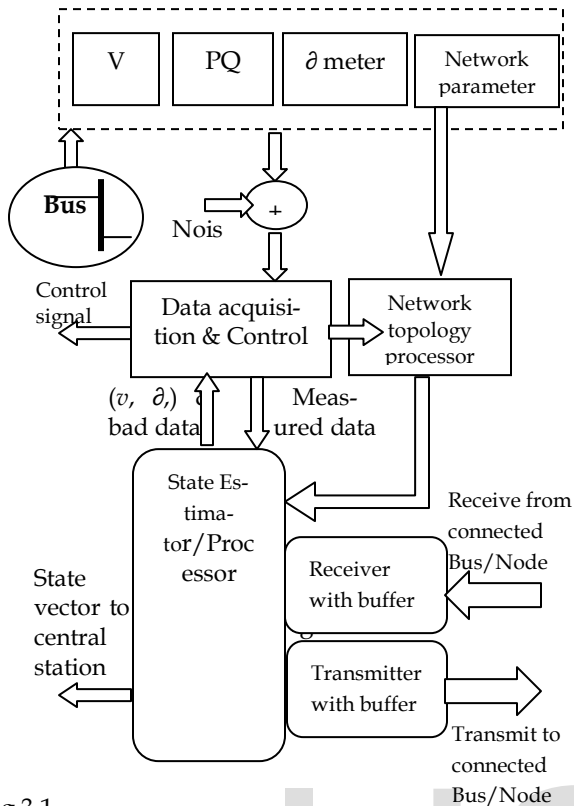


Fig.3.1

3.3 Flow chart

The fig 3.2 represents the 'kth' node along with its connected Node; The Fig 3.3 represents the 'kth' Node Area computational flow diagram.

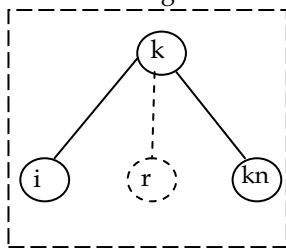


Fig.3.2

Note:- Super script 'tr' and 're' stands for transmitted & received value respectively.

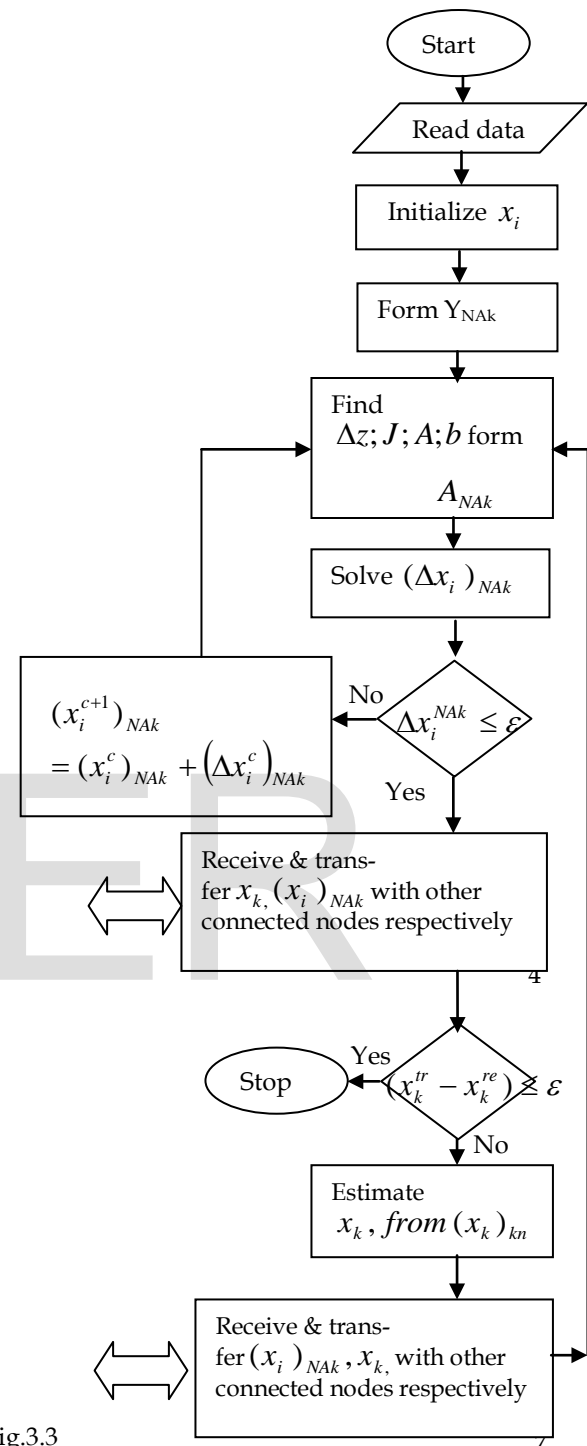


Fig.3.3

4 EXAMPLE &RESULTS

The above concept is tested on a 13 bus test system for ISE and NASE. The estimation results of both the methods are found to be same up to four decimal. The results of computational time are tabulated in the following section. In both the methods (ISE and NASE) proper indexing is used to avoid non-zero computations. Or in other words all the computations are focused only on non-zero elements without using any "if" statements.

4.1 Input tables (note – all the quantities are in pu)

TABLE 4.1.1 -Line data

Node-i	Node-j	r	x
1	2	0.00148	0.0028676
2	3	0.000438	0.00124174
3	4	0.000277	0.00078488
4	5	0.000598	0.00166769
4	8	0.0016	0.00310017
5	6	0.000343	0.0009719
5	9	0.000343	0.0009719
6	7	0.000324	0.00091669
7	10	0.000324	0.00091669
8	9	0.000294	0.0008338
9	10	0.000532	0.00150562
9	12	0.000378	0.0010705
10	11	0.000588	0.00166488
11	13	0.000324	0.00091669
12	13	0.000368	0.00104355

TABLE 4.1.2 -Injected power, Voltage & angle measurement

Bus No.	Pi	Qi	Vi	δ_i
1	28.162	13.18	1.053269	--
2	-5.89546	-1.07074	--	--
3	-3.42084	0.330078	0.958231	-0.08616
4	-2.47793	0.706497	--	--
5	-6.9646	-0.50879	0.928889	-0.12301
6	-4.15897	0.041931	--	--
7	-2.5218	0.616272	--	--
8	-6.13422	-0.9848	--	--
9	-4.89722	0.894775	0.925278	-0.12723
10	-2.59491	0.689423	0.923398	-0.13011
11	-5.998	-0.86444	0.919737	-0.13517
12	-3.93211	-0.04755	--	--
13	-2.38309	0.562073	0.920707	-0.13377

TABLE 4.1.3 -Line flow measurements

NA	Node i	Node j	pij	qij
1	1	2	28.5327	13.0812
2	1	2	28.5327	13.0812
2	2	3	22.4874	8.978
3	2	3	22.4874	8.978
3	3	4	20.9456	7.7882
4	3	4	20.9456	7.7882
4	5	4	-12.823	-4.76946
4	4	8	7.53516	2.11797
5	5	4	-12.823	-4.76946
5	5	6	4.02863	1.43748
5	5	9	4.40723	1.90301
6	6	5	-4.02136	-1.41695
6	6	7	1.90622	0.733363
7	6	7	1.90622	0.733363
7	7	10	1.4794	0.60305
8	8	4	-7.42565	-1.90575
8	8	9	2.72708	0.362727
9	9	5	-4.39807	-1.87707
9	9	8	-2.72452	-0.35546
9	9	10	1.81613	0.516504
9	9	12	4.02228	1.30623
10	10	7	-1.47842	-0.60032
10	10	9	-1.8139	-0.51024
10	10	11	2.93549	1.00072
11	11	10	-2.92885	-0.98192
11	11	13	-1.45929	-0.45715
12	12	9	-4.01439	-1.2839
12	12	13	1.89218	0.59386
13	13	11	1.46017	0.459604
13	12	13	1.89218	0.59386

Note: - The line measurements v_i and δ_i are duplicated at the node-area but for ISE it is not necessary.

4.2 Result tables

TABLE 4.2.1 – ISE & NASE results (13 bus test system)

Number of iteration = 3.

Bus No.	V-ISE	∂ -ISE	V-NASE	∂ -NASE
1	1.05328	0	1.053265	0
2	0.979382	-0.0605903	0.979367	-0.060595
3	0.958245	-0.0861526	0.958232	-0.086156
4	0.945926	-0.101909	0.94593	-0.101913
5	0.928904	-0.123005	0.928908	-0.123005
6	0.925919	-0.126983	0.925930	-0.127000
7	0.924528	-0.128747	0.924530	-0.128760
8	0.926483	-0.1247	0.926480	-0.124700
9	0.925293	-0.127228	0.925297	-0.127230
10	0.923413	-0.130107	0.923407	-0.130110
11	0.919752	-0.135169	0.919744	-0.135170
12	0.922148	-0.131696	0.922136	-0.131700
13	0.920722	-0.133764	0.920710	-0.133770

TABLE 4.2.2 –NASE results – Node Area Wise (Voltages)

V _i	v(pu)	Avg.
V ₁	1.05326 (1); 1.05327(2)	1.053265
V ₂	0.979362(1); 0.979365(2); 0.979375(3)	0.979367
V ₃	0.958228(2); 0.958238(3); 0.958232(4)	0.958233
V ₄	0.945919(3); 0.945913(4); 0.94595(5); 0.945938(8)	0.94593
V ₅	0.92889(4); 0.928928(5); 0.928919(6); 0.928897(9)	0.928909
V ₆	0.925943(5); 0.925935(6); 0.925907(7)	0.925928
V ₇	0.924544(6); 0.924516(7); 0.924529(10)	0.92453
V ₈	0.926469(4); 0.926495(8); 0.926476(9)	0.92648
V ₉	0.925317(5); 0.925305(8); 0.925286(9); 0.925294(10); 0.925281(12)	0.925297
V ₁₀	0.923401(7); 0.923406(9); 0.923414(10); 0.923405(11)	0.923407
V ₁₁	0.919753(10); 0.919744(11); 0.919736(13)	0.919744
V ₁₂	0.922141(9); 0.922136(12); 0.922132(13)	0.922136
V ₁₃	0.920714(11); 0.92071(12); 0.920706(13)	0.92071

TABLE 4.2.3 –NASE results – Node Area Wise (Angles)

∂	rad	Avg.
∂_1	0	0
∂_2	-0.060593(1); -0.060592(2); -0.060601(3)	-0.060595
∂_3	-0.086155(2); -0.086163(3); -0.08615(4)	-0.086156
∂_4	-0.10192(3); -0.101907(4); -0.101913(5); - 0.101913(8)	-0.101913
∂_5	-0.123002(4); -0.123008(5); -0.123008(6); -0.123003(9)	-0.123005
∂_6	-0.127(5); -0.127(6); -0.127(7)	-0.127
∂_7	-0.12875(6); -0.12877(7); -0.12875(10)	-0.12876
∂_8	-0.1247(4); -0.1247(8); -0.1247(9)	-0.1247
∂_9	-0.12723(5); -0.12723(8); -0.12723(9); - 0.12723(10); -0.12723(12)	-0.12723
∂_{10}	-0.13013(7); -0.13011(9); -0.13011(10); - 0.13011(11)	-0.13011
∂_{11}	-0.13517(10); -0.13517(11); -0.13518(13)	-0.13517
∂_{12}	-0.1317(9); -0.1317(12); -0.1317(13);	-0.1317
∂_{13}	0.13377(11); -0.13377(12); -0.13377(13)	-0.13377

Note: - The number inside the bracket represents the Node Area. For example '9' means viewed from Node Area 9 or NA9.

TABLE 4.2.4 –Computational time

Sl No.	Method	Comp time (ms)
1	ISE	2.640
2	(NASE) _{max}	0.290
3	(NASE) _{avg.}	0.179

Note: - (NASE)_{max} is observed at node 9, because the node 9 has maximum connected nodes.

TABLE 4.2.5 –Relative time

Sl No.	Method	Comp time Ratio
1	ISE/(NASE) _{max}	9.103
2	ISE/(NASE) _{avg.}	14.74

Note: - Above timings are obtained using profiling tool. These timing are also dependent on the processor and the operating system.

5 CONCLUSION

The size of the network increases with the increase in number node/buse. But whatever may be size of the network, the node having maximum number of connected nodes does not depend on the total number of node/bus in the network. For

example let us assume a network having a 'node' with maximum connected nodes will be, say ≈ 12 , then the time taken to solve using 'n' number of processors is \approx time taken by the node area having 12 connected nodes (NA_{mx}). In the above 13 bus test system node number 9 has the maximum number of connected nodes, hence the time taken by NA_{13} can be considered as actual NASE estimation time. It is evident from results that regardless of size of network the computation can be completed within mili-seconds using NASE technique.

The state of each bus/node depends on the status of its neighbor (connected bus/node). The NASE technique makes use of 'a' node/bus along with its neighbor to estimate its state variables. It is evident from the results that the NASE is a good solution for large size, complex state estimation. The NASE technique divides the single complex problem into multiple independent smaller sub-tasks which are independent at Node Area level. The number of state variables to be computed at each Node Area is also very small as compared to the whole network. Hence, independent processors can be easily employed at each Node to solve the single large complex problem more easily without complicated parallel processing architecture. A bus/node can be made smart using NASE. The disadvantage of this technique is increase in the measurements because at each node area sufficient measurements should be made available to obtain reliable results. The said disadvantage can be compensated by carrying out state estimation only at selected nodes area. These possibilities are given encouraging results which will be published in the next paper.

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7 BIOGRAPHIES



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